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Zimbabwean High School Teachers' Interpretations of Learners' Alternative Conceptions on Selected Baseline Test Items on Calculus and Trigonometry Concepts

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Abstract: This study reports the alternative conceptions that were exposed by high school learners on some concepts on limits and trigonometry. The alternative conceptions were discussed at a day-long teacher professional development workshop organised for high school teachers teaching in Nyanga district, Zimbabwe. The deliberations of the workshop were video taped in order to allow replays during data interpretation. The sixteen teachers who attended the workshop held different conceptions on the nature of mathematical knowledge and skills which influenced their assessment of the learners' alternative conceptions. However, the teachers concurred on the interpretations and professional insights that they developed from the learners' alternative conceptions. The use of baseline-test performance to inform teaching practice is presented as an effective way of diagnosing learners' prior knowledge that teachers can use to facilitate learners' development of new concepts.

Key words: Alternative conceptions; prior knowledge; conceptions; formative assessment

Introduction

In student-centred learning environments, learners are not perceived as empty of mathematical knowledge waiting to be filled (Davis, 1990). In such learning environments learners are believed to bring to mathematics classes formal and informal ideas, skills, attitudes, and beliefs that can promote or hinder their learning. Successful student learning of mathematical concepts happens when teachers correctly estimate learners' current knowledge systems and are able to increase the learners' psychomotor skills, intellectual capacities or both. Using the estimated prior knowledge, teachers can organize appropriate activities that can facilitate learners' understanding of new concepts. The assumed prior knowledge and procedures that learners sometimes hold concur with formal mathematical procedures or may not. When learners apply their prior knowledge and use procedures that are different from formal mathematical processes that lead to different solutions, teachers often assume that the learners hold some misconceptions or alternative conceptions. Misconceptions arise from learners'

erroneous or illogical use of mathematical procedures and processes that leads to solutions and products that are different from those obtained when using the logical and correct procedures. Alternative conceptions, on the other hand, are taken in this study as learners' world views of procedures for finding solutions to mathematical problems that are contextually framed and logically consistent but lead to solutions that are the same or different from the formal procedures for solving similar problems on a given topic.

It is important that teachers solicit and carefully interpret the ideas and conceptions that learners bring to mathematics classes so these can be worked with and built upon during learners' construction of new understanding (Setati, 2002). Schelfhout, Dochy, Janssens, Struyven and Gielen (2006) advised that by carefully selecting some mathematical tasks at the beginning of a learning episode, teachers can test learners' thinking in ways that can facilitate exposition of the learners' misconceptions and alternative conceptions. This may provide the teachers with foresight to interpret the learners' current mastery of knowledge and skills on concepts that they are to learn. Learners' current understanding of concepts to be covered in a lesson can be solicited through baseline-test assessment. Engaging learners in cognitive conflicts in baseline-test items and later discuss with them the possible conflicting view points, helps teachers to see how and why some learners' ideas need to be strengthened or altered in order for them to reconstruct their understanding and master related concepts they are to learn. Baseline-test items that expose learners' challenges on concepts they are about to learn have potential to facilitate teachers' instructions that can enable the learners to build new concepts from their prior knowledge.

Effective teaching of mathematics is a complex and interpretive enterprise (Even, 2005) that demands teachers' reflective actions during teaching in order to appropriately respond to students' learning needs. Formative assessments in which teachers determine students' learning needs through question and answer sessions or observation of mastery of mathematical skills have been used to inform teaching practice in limited ways (Black, 1998; Schelfhout et al., 2006). There is a scarcity of literature on teachers' interpretations of learners' alternative conceptions on the concepts of limits and trigonometric functions exposed in baseline-tests administered before teaching the respective topics. Furthermore, Harrison (2006) calls for detailed research that considers teachers' interpretations of learners' alternative conceptions. This study is a response to such calls. The study hopes to contribute to the understanding of mathematics teachers' interpretations of learner alternative conceptions and how the teachers can utilize the interpretations to improve their instructional practices. The learners' alternative conceptions were exposed on self-written solutions of baseline-test items on some selected high

school calculus and trigonometry concepts. In Zimbabwe high school classes are attended by students in the age range of 17 to 18 attending Form Five or Six (Grade 12 or 13).

The study was guided by the research question: What professional insights can high school mathematics teachers draw from their interpretations of learners' alternative conceptions exposed on solutions of baseline-test assessment tasks on the topics of limits and trigonometry? The answers to the research question are hoped to contribute a theory that informs teachers on how to elicit, interpret, and utilize learners' prior knowledge that they bring to mathematics classes in order to improve their instructional practice. Awareness of the knowledge, ideas, and skills that learners sometimes bring into mathematics classes may give teachers insight into organising classroom activities that have potential to strengthen or enable learners to revise their current understanding of some mathematical concepts in order to accommodate and assimilate new ones.

Learners' Alternative Conceptions: A Theoretical Perspective

Teachers' work, in part; consists of initiating, establishing, and monitoring the ternary relationship between the teacher, the learners, and the concepts at stake (Steinbring, 2005). An entirely value-free and conception-less understanding of teachers' interpretations of the relationship between learners' methods for solving mathematical problems and understanding content at stake is not possible (Clarke, Breed & Fraser, 2004; Nyaumwe, 2004; Steinbring, 2005; Webb, 2005). Teachers' interpretations of the relationship between learners' mathematical ideas and content are explicit in how questions, answers, views, and controversies are addressed. Teachers' beliefs on how learners express and learn mathematical concepts can be interpreted using a continuum ranging from absolutist to fallibilist view-point (Ernest, 1989; Webb, 2005). Characteristics of teachers' views of the nature of mathematical concepts on each end-points of the absolutist–fallibilist continuum are briefly outlined next.

Teachers with instructional beliefs belonging to the absolutist continuum view mathematical content as static, fixed or sacrosanct. They believe that sources of legitimate mathematical processes are correct applications of axioms, definitions and theorems (Davis & Hersh, 1981). The products of mathematical activities in the absolutist continuum arise from logical mechanistic use of formal accepted procedures. As a result of the correct use of formal rules, mathematical truths are perceived as unquestionable, certain, and objective. Teachers holding absolutist views of the nature of mathematical procedures and products usually use teacher-centred learning environments where drill and practice approaches are the dominant

methods used in the teaching and learning of mathematics. Teachers holding opposing views to the absolutist continuum hold instructional views that belong to the fallibilist continuum.

Teachers holding fallibilist views of the nature of mathematical concepts believe that mathematical knowledge can be individually or socially constructed by learners through observations, experimentations, and abstractions using senses and can therefore be fallible (Davis, 1990; Webb, 2005). This makes such teachers believe that mathematical knowledge is context based, tentative, intuitive, subjective, and dynamic because it can be revised and corrected (Ernest, 1989). Such teachers believe that sources of learners' mathematical strategies are intuition, creativity, pattern searching, and trial and error among others. As such, mathematical solutions cannot be pre-determined using universally accepted algorithms as they depend on the context used and how individuals interpret the tasks. Incompatibility or compatibility of teachers' and learners' solutions is attributed to constraints and opportunities presented by the social contexts in which the problems were generated and interpreted. Solutions to mathematical problems are not unitary but multiple, depending on the context that is used to approach a problem. Teachers with views belonging to the fallibilist continuum usually create student-centred learning environments that encourage learners to individually or with peers collectively construct mathematical relationships and knowledge.

Interpretation of learners' intuitive applications of their ideas and methods of solving mathematical problems is influenced, in part, by the instructional conceptions that a teacher holds. In the same way, teachers assess learners' mathematical solutions using their personalized frames of the nature of mathematical knowledge that they hold. Assessment is used here to mean the activities that a teacher sets to find out learners' current understanding of some mathematical concepts (Schelfhout et al., 2006). Through assessment, learners' prior experiences, knowledge, and alternative conceptions that are relevant to the learning of content on a topic are determined. Teachers with views belonging to the absolutist continuum expect learners to provide unequivocally correct solutions using formal correct step-by-step applications of axioms, theorems, and algorithms that produce solutions that concur with pre-determined answers. On the other hand, teachers with views belonging to the fallibilist continuum expect learners to logically reason with the constraints presented in mathematical problems. The learners are expected to use their intuitions to invent possible solution strategies that are based on the internal consistency of the structure of mathematical concepts used. Learners' solutions in this paradigm are not pre-determined, but can vary from one learner to another, depending on how each learner interprets the problem and its context.

Various formative assessment methods that reveal students' learning needs and strengths are used by teachers to modify learning programmes in order to offer effective teaching. Some of the methods used are baseline-tests, oral questions, written exercises, and post tests. Baseline-tests are written under examination conditions before students learn a new concept. They are conducted to assess the ideas and skills that learners bring with them that can be used to build new concepts. Oral questions are posed during instruction and learners answer them as a lesson progresses. Written work is usually given in the form of class-work or home-work and submitted for teacher assessment. Learners write post-tests after covering the depth and breadth of content on a topic to the expectations of the level's curriculum demands.

This study analyses learners' alternative conceptions, how teachers interpreted them and the professional insights that the teachers developed from the alternative conceptions. Findings from the study are hoped to develop, in teachers, an appreciation that inquiry into learners' alternative conceptions is important for improving teaching. The alternative conceptions that learners sometimes show on baseline-test items of some topics that they are to learn, reveals their current knowledge of mathematical concepts. Such current understanding may need strengthening or revising in order to concur with contextual or formal mathematical understanding. Knowledge of learners' entry understanding of mathematical concepts on a topic they are to learn, may give teachers insight into organising appropriate learning activities that may facilitate the learners' use of their prior knowledge to construct new knowledge on the topic. The context described below was used in conducting this study.

Context of the Study

The learners' alternative conceptions that are discussed in this paper were compiled by high school teachers from their learners' solutions of baseline-test items over a period of time. Construction of the baseline-test items was based on the understanding that learners possessed some prior knowledge that they could use to develop new content on a topic.

The baseline-tests contained some items on content that learners covered earlier in the spiraled mathematics curriculum and others that they were to learn on a topic. The learners were expected to answer items covering new content that they were to learn on a topic using their intuitive knowledge or understanding from own advance reading on a topic. The teachers asked learners to write baseline-tests without prior announcement as they were expected to expose their current understanding of

concepts to be covered on a new topic. The purpose of writing baseline-tests was to determine the aspects of a new topic that learners already knew in order to adjust the pace and level of difficulty of concepts on a topic. When interpreted correctly, baseline-test results have potential to facilitate teachers to pitch concepts as closely as possible to the cognitive understanding of individual learners. The design that was used in conducting the research is presented next.

Methods

This researcher was invited to facilitate at a day long Science Education Inservice Teacher Training (SEITT) workshop organised for high school teachers in Nyanga district. The district is located in the eastern part of Zimbabwe. The SEITT programme was initially introduced to enable peer graduate high school teachers in the same geographical confinements to staff develop each other in order to improve the quality of their teaching. SEITT resource teachers who received two-year part-time training at the University of Zimbabwe provide "in-service education of their peers through, among other things, organizing and running subject centred workshops for teachers" (Mtetwa, 2003, p. 77). The mathematics workshop under discussion was attended by 16 participants. The workshop was organised to assist mathematics teachers based in Nyanga district to analyse learner misconceptions, interpreting them and subsequently utilize the earners' alternative conceptions in their teaching.

The focus of the workshop was on seven alternative conceptions that learners exposed on their solutions on some baseline-test items. The seven alternative conceptions were drawn by teachers from various pure mathematics concepts of logarithmic differentiation, Pythagoras theorem, limits, vectors and calculus. The learners' alternative conceptions were forwarded to the SEITT district centre where they were compiled. Seven alternative conceptions were perceived as a number large enough to occupy the teachers on a day-long SEITT staff development session on the theme "Interpreting and utilizing learners' alternative conceptions during teaching". Learners' solutions on two of the seven alternative conceptions that were deliberated on in the workshop raised heated debates among the teachers on the status of the learners' solutions. The teachers agreed to disagree on the status of the two alternative conceptions. The debate on question four on limits and question six on trigonometry (arcsine) was polemic in that more time was spent on them than the other five learners' solutions.

The polemic issues that were generated by the learners' alternative conceptions on these two questions demanded further scrutiny in order to analyse the exposed learners' thinking. The analysis of learners' thinking on these two solutions had

potential to help teachers to interpret the alternative conceptions in detail. The present paper is based on analyses done on these two polemic questions in order to theorize the teachers' interpretations and how they could utilize the alternative conceptions to enhance development of the learner understanding of the procedures and other related concepts.

The deliberations of the day-long workshop were video taped for the purposes of playing back for data analysis for this study. Replays of the video tapes provided in-depth understanding of the teachers' reasoned arguments.

For ethical reasons and to protect the identity of the 16 participating teachers in this study, the teachers are only identified by numbers representing their sitting positions during the workshop. The teachers were seated in a classroom in rows facing the blackboard. The seating arrangement facilitated all participating teachers to see the selected learner's alternative conceptions on a baseline-test item reproduced on the chalk-board. This seating arrangement enabled participating teachers to discuss the focal alternative conception in full view of the learner's steps in the solution. The sitting arrangement also made visibility of illustrations that teachers were capable of making on the chalk-board to interpret and utilize the learner's alternative conception in their teaching practice.

Eleven of the participating teachers were male whilst five were female. All the teachers were teaching high school mathematics classes. Nine of the participating teachers hold Bachelor of Science degrees with mathematics as either a major or minor subject at degree level. Seven of the teachers hold Bachelor of Education degrees with a specialization in mathematics. Of the nine teachers holding Bachelor of Science degrees, six had qualifications in education and the remaining three teachers were teaching without professional qualifications. The teachers' teaching experiences ranged from three to fourteen years of teaching service with mean teaching years of eight. The teachers' bio data are typical of mathematics teachers in the country where gender distribution in science related subjects is larger for males. In the country, a teacher holding a first degree only can teach without professional qualifications and high school mathematics teacher turn over is high.

Assuming that relevant qualifications translate to competent teaching, the participating teachers possessed mathematical knowledge that was sufficient to handle high school content. A mean of eight years of teaching experience was satisfactory to enable the teachers to interpret learners' alternative conceptions from theoretical as well as practical experience of how learners solve mathematical problems in natural classrooms. A template of the question item, a learner's solution, teachers' interpretations of the solution, the professional insights

developed by the teachers, and the status of the solution is used. The results of the study are presented question by question below.

Results

Results from the first question

In the first question, learners were given an example of finding the limit of an algebraic fraction. They were to use their understanding of the example to find the limit of a different function as shown below:

Given

$$\lim_{x \to 8} \left(\frac{1}{x - 8} \right) = \infty$$

Find

$$\lim_{x \to 5} \left(\frac{1}{x - 5} \right)$$

Student's solution:

$$\lim_{x \to 5} \left(\frac{1}{x - 5} \right) = 5$$

(Source: Teacher 9)

Teachers' interpretations of the learner's understanding

Most of the participating teachers concurred that the learner's solution was influenced by the example given. Supportive of this claim are the following verbatim statements; "the student considers ∞ as 8 rotated through a right angle in an anti-clockwise direction and took the principle for limit as x approaches a given number as that number rotated, and rotates 5" (Teacher 8). Another interpretation was that "…the learner's solution is influenced by the pattern that exists in the example" (Teacher 13). Some interpretations were leveled against the limited examples given in the question. Such teachers argued that "the learner's solution is influenced by limited examples. Several examples could have given the learner some images from which to make a generalization" (Teacher 1).

Professional insights developed by the teachers

The participating teachers developed some instructional insights from analyzing the learner's solution. They suggested several strategies to assist the learner when

teaching the concept. They suggested that learners' understanding of the concept of a limit could be developed from substituting numbers close to the limiting number. For instance "…to introduce the conceptual meaning of a limit as x approaches 8, values close to 8 from below such as 7.9; 7.99; 7.999; 7.9999… and those approaching 8 from above such as … 8.001, 8.01, 8.1 can be substituted in the function in order for learners to study the behaviour of the resultant values" (Teacher 5). Teachers with such views argued that "when learners have seen the effect of dividing one by a number that is very small, they may understand the concept of a limit as a number that is very close but not the limiting value itself. This may be used to introduce division by very small numbers that are close to zero which in turn can be effective to introduce asymptotic lines" (Teacher 16). The other teachers proposed that a graphical representation of the function could enable learners to study the points on the graph close to the limiting value. They argued that, "a graphical illustration of the concept of a limit may enable learners to see the behaviour of the function when values of x get very close to 8 from below and from above. The graphical illustration may facilitate learners' visualization of the concept of infinite using asymptotic lines. This may help the learners to understand the concept of a limiting value" (Teacher 10).

Some teachers believed that the example given was a poor one and was limited in scope to a situation that could be interpreted differently by learners. The example influenced the learner's solution strategy arguing that "the teacher should have avoided using an example that shows some patterns. Distinct examples like

$$\lim_{x \to 2} \left(\frac{1}{x-2} \right) = \infty$$

could be clearer to learners" (Teacher 9). After discussing the teachers' interpretations and professional insights that they developed from the learner's alternative conceptions, it was necessary to discuss the status of the solution. This discussion is presented next.

Assessment of the status of the learner's solution

The participating teachers were divided over the status of the learner's solution. The participating teachers who totally rejected the solution as wrong argued that, "the limit was not calculated; therefore, the solution is wrong" (Teacher 4). "The solution is illogical in that some procedures that are not applicable to limits were used" (Teacher 8). A third opinion against the solution was "the solution is wrong because a mathematical problem has a unique solution that cannot be negotiated. If students are to pass public examinations they should be assessed using the standard that is used for their summative evaluation" (Teacher 7).

The participating teachers who held contrary ideas felt that the learner's solution should be considered to be viable under the context that the learner interpreted the question. Their argument was that "the solution should be considered correct because the student applied the concept of rotation correctly" (Teacher 3). In support of Teacher 3, Teacher 12 said "the student was confronted by applying concepts never encountered before and used the given example to logically reason that limits involve rotations". Teacher 11 reinforced the argument saying "in view of the unclear example, the student should be marked right because s/he used patterns correctly to find a solution to the problem".

On the basis of their individualized conceptions of the status of mathematical solutions, the teachers agreed to disagree on the status of the learner's solution.

Results from the second question

This question required factual understanding and applications of knowledge on trigonometrical functions. The question was stated as:

Solve for *x*, in

$$y = \sin^{-1} x.$$

Learner's solution:

$$y = \sin^{-1} x \Rightarrow y = \frac{1}{\sin} x,$$

$$\therefore \sin y = x.$$

(Source: Teacher 11)

Teachers' interpretations of the learner's understanding

The participating teachers interpreted the learner's solution and concurred that the learner used an algebraic understanding to solve a trigonometrical problem that require different applications of procedures. This claim is based on the teachers' statements such as; "the learner understood the concept of algebraic inverse as also applying to trigonometrical ratios" (Teacher 6). This line of thinking was further elaborated by Teacher 13 who argued that "the student's understanding of arcsin is the same as that of inverse in algebra i.e. thinks that $\sin^{-1} x = \frac{1}{\sin} x$. This understanding, however, is not correct in trigonometry." The participating teachers drew the below mentioned professional insights from the learner's alternative conceptions.

Professional insights developed by the teachers

The teachers unanimously agreed that the learner has correct understanding of the inverse concept for algebraic functions. In support of the learner's correct algebraic understanding of brackets Teacher 5 argued that "in algebra $x^{-1}y = \frac{y}{x}$. Using his/her current understanding, the learner simplifies $\sin^{-1}x$ to $\frac{1}{\sin}x$." The participating teachers argued that the learner's algebraic understanding should be different from the trigonometrical understanding where inverse is defined as 'arc'. In support of the contextual meaning of inverse ($^{-1}$), Teacher 14 echoed the sentiments expressed by others "the definition of inverse in trigonometry is different from the algebraic definition. In trigonometry $\sin^{-1}x$ is read as arcsinx and not the inverse of sinx."

The participating teachers agreed that in order to help learners to review the alternative conceptions expressed in the learner's solution "the concept of arcsinx can be introduced using learners' prior knowledge on special angles. For instance, an exact example such as $\sin 30^{o} = \frac{1}{2} \Rightarrow \sin^{-1}(\frac{1}{2}) = 30^{o}$ can be used to show learners the different definitions of the sign $^{-1}$ in algebra and trigonometry. "A generalization of this exact example in trigonometry can be shown as $\sin y = x \Rightarrow y = \sin^{-1}x$" (Teacher 5). Using the insight that they developed from their interpretations and the professional insights they developed from the alternative conception, the participating teachers deliberated on the status of the solution as presented in the next section.

Assessment of the learner's solution

The participating teachers consensually agreed that the solution was correct but the understanding that was used by the learner was flawed. A polemic discussion was then centred on whether or not full credits can be given to the learner for getting a correct solution from wrong working. The participating teachers' opposing arguments are summarised by verbatim statement that follow. Some participating teachers proposed that the learner gets full credits for the solution because "s/he correctly used prior knowledge of algebraic inverse functions. The learner has applied that knowledge correctly in solving related problems in trigonometry, without breaking any mathematical procedures" (Teacher 3). Opposing this view some participating teachers who considered the procedure used in getting the solution as incorrect expressed sentiments summarised by Teacher 7 who argued that "the student's solution cannot get credit because the understanding shown in the

solution contradicts with the understanding expected in trigonometry. The procedure used to get the solution is wrong".

The teachers' line of reasoning seemed to be influenced by their conceptions of mathematical solutions. Those who view mathematical solutions as arising from formal applications of correct procedures in the branch of mathematics viewed the solution as incorrect. Those teachers who believed in the promotion of learners' logical reasoning and consistent application of mathematical procedures viewed the solution as viable.

Discussion

The high school mathematics teachers who attended the workshop on learners' alternative conceptions developed a lot of professional insights from the learners' solutions. The learners' solutions to the baseline-test items revealed to the teachers that learners bring knowledge and skills to the learning environment that teachers should pay careful attention "in order to use that understanding to develop new concepts" (Setati, 2002, p. 9). Whilst the participating teachers failed to agree on the status of the solutions that some learners produced, they agreed that the solutions certainly had no misconceptions but showed some learners' alternative conceptions on solving them.

The professional insights that the teachers developed from the learners' alternative conceptions were influenced by the instructional conceptions that they hold about the ternary relationship between the teacher, the learner and the mathematical content. The ternary relationship was smooth and straight forward for participating teachers who valued teacher-centred learning environments. This was the case because learners' solutions to baseline-test items in teacher-centred environments are either correct or wrong depending on the procedures that the learners use and the solutions that are produced (Steinbring, 2005). However, learners' solutions of baseline-test items in student-centred learning environments are not out-rightly correct or wrong. Learners in such classes independently construct mathematical knowledge and procedures using personalized understanding and interpret questions at hand using contexts familiar to them (von Glasersfeld, 1991). Learners' active construction of meaning using their personal contextualized understanding to make sense of mathematical problems using the social norms of inquiry, reasoning, argumentation, and intellectual autonomy require that learners' solutions be assessed using a multiplicity of approaches that are justifiable and explained (Even, 2005). For instance, the contextual interpretation of a question by learners who are not exposed to the arcsine concept in trigonometry should not disadvantage the

learners who correctly use the algebraic inverse notation in solving a trigonometrical equation (see arguments for assessment of the second question).

The argument for accepting a learner's solution to the second question is presented as follows: In assessing learners' baseline-test solutions, teachers use privileged knowledge of the direction that a topic is to take, or the skills that learners are to develop on a topic. According to Steinbring (2005) such knowledge makes teachers to use content related analyses of learners' responses to items in a baseline-test. This enables the teachers to expect learners to exhibit certain predetermined procedures and skills. This expectation is contrary to learners' autonomous interpretation of questions using frames that may differ from those expected by their teachers. Learners who demonstrate understanding of solving a baseline-test question item using a method different from that expected by a teacher should be assessed on the basis of common sense as well as mathematical sense based on logical arguments presented, whether such alternative thinking produces identical or different predetermined solutions (Martinez, 2001).

In the teacher-centred learning environment, variance of a learner's solution strategy from that of a teacher is often taken to mean misconceptions in a learner's understanding. However, in the student-centred learning environment that recognize that mathematical concepts are context-based, variance may be viewed as a learner's alternative conception that leads to different interpretation and understanding. This may lead to the use of different approaches that give rise to different solutions. In such classrooms mathematical problem-solving do not focus on the status of solutions whether they are right or wrong, but concentrate on the viability of the context(s), reasoning and method(s) used to solve the problem. Assessment of learners' solutions in such classrooms focus on formulating and evaluating the viability of the reasoning, evidence, examples, and arguments presented by learners in finding solutions to a problem.

Another key element that was instrumental to the teachers' failure to agree on the status of learners' alternative conceptions was the intended purpose of baseline-test items. When teachers set assessment tasks, they should consistently bear in mind what they want to measure and to infer. Whilst all assessment methods are used on the assumption that learners should have full opportunities to demonstrate their understanding and ability to perform the tasks at hand, the purpose of formative assessment to guide teaching decisions and actions to be taken determines the assessment type to be used by a teacher. Commonly, tests probe for learners' understanding, reasoning, and utilization of already mastered concepts through learner recall and application of mathematical facts, concepts, conjectures and theorems. In such tests learners are expected to use uniform procedures that bring

out predetermined solutions. The ability of a learner to correctly use the predetermined procedures reveals to teachers a broad range of capabilities and understanding of the mathematical concepts that are mastered by the learners (Lowery, 2003). Otherwise failure to use formal procedures for solving certain types of questions is a sign of lack of understanding of the rudiments of the concepts of a topic.

Unlike the usual tests, baseline-tests diagnose learners' levels of mastery of content and procedures that they are about to learn on a new topic. To expose the prior knowledge that can be utilized in the learning of new concepts of a topic learners use their informal and formal reasoning, creative thinking, and ingenuity to explore patterns in their search for logical solutions. The understanding that learners expose in finding solutions to baseline-test items informs teachers of the relevant knowledge that they can strengthen or adjust in order to develop concepts on a new topic with understanding. To encourage learners to develop inquiry, logical thinking, and reasoning skills, teachers should encourage learners to expose their prior knowledge through assessment of learners' alternative conceptions by rewarding evidence of using common sense and mathematical sense making in solving baseline-test items. This may nurture learners' curiosity to logically invent and construct viable mathematical procedures.

The tension between summative and formative evaluation was an obstacle that influenced some teachers' assessment decisions on learners' alternative conceptions (Teacher 7). For instance, in assessing the viability of a learner's alternative conception to the first question, Teacher 7 argued that "if students are to pass public examinations they should be assessed using the standard that is used for their summative evaluation". This statement illustrates that some teachers' pre-occupation with ideas to coach learners for summative examinations often overshadow their purposes for formative classroom assessments. As also noted by Black (1998) the tensions that teachers often face between summative and formative evaluations make some of them perceive assessment of learners' alternative conceptions in formative evaluations as out rightly wrong or right without probing the understanding proffered as they treat them in the same manner that they treat standardised test solutions.

Conclusion

Baseline-tests remain a viable barometer that measures learners' prior knowledge on a new topic. Teachers' interpretations of the learners' performance on baseline-tests provide them with insight to alter their classroom programmes in order to reinforce learners' strengths and address knowledge gaps that learners may expose in their

learning. To promote reform on learner-centred instruction, in their assessment of learners' alternative conceptions in baseline-tests items, teachers should not view mathematical problems as unproblematic for all to understand, but accept that they can be understood differently by individual learners. As such, mathematical problems in baseline-test items are not endowed with universal meaning but rather derive their meaning from the way in which individual learners attend to them using personalized understanding and contexts. To appreciate learners' alternative conceptions exposed on solutions of baseline-tests, some teachers may need to review their professional obligations of coaching learners for summative evaluations, insist on learner use of formal notations and procedures, and prestige of method to solve problems. Reviewing such beliefs may facilitate some teachers to create classroom environments that encourage learners to make sense of mathematical problems in ways that enable them to expose multiple methods for solving them without focusing on whether their solutions are right or wrong.

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